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The extension of self-avoiding random walk series in two dimensions

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Abstract. We have extended the enumeration of square lattice and triangular lattice self-avoiding walks and their end-to-end distance by two terms. We have also calculated 17 terms in the series for the resistance of SAWs with bridges on the square lattice. Analysis of the new data allows the connective constants to be more accurately estimated, and affirms the values $\gamma = \frac{43}{32}$ and $\nu = \frac{3}{4}$, and $1+\gamma$ for the exponent of the resistance series.

1. Introduction

In an earlier paper (Wang 1989) we presented an algorithm for enumerating SAWs. We subsequently found that this algorithm had in fact previously been developed by Torrie and Whittington (1975). In a related paper, Guttmann (1987) gave a systematic method for the analysis of such series based on the method of differential approximants (for a full description see Guttmann 1989a).

In this paper we report the extension of the SAW chain generating function and mean square end-to-end distance series for the square and the triangular lattice data, both by two terms. Thus we now report data to 22 and 29 terms on the triangular and square lattices respectively. The calculations were performed on a Masscomp 5700, and took about 700 hours for the triangular lattice calculation, and somewhat less for the square lattice calculation.

By similar methods we have enumerated the mean resistance averaged over all self-avoiding walks with bridges. This series is given to 17 terms for the square lattice, and augments the related work on this problem by Manna *et al* (1989).

The new terms are:

Square:	$c_{28} = 2\,351\,378\,582\,244$	$c_{28}\rho_{28}/4 = 69\,477\,665\,745\,896$
	$c_{29} = 6\,279\,396\,229\,332$	$c_{29}\rho_{29}/4 = 195\,265\,123\,427\,301$
Triangular:	$c_{21} = 32\,944\,292\,555\,934$	$c_{21}\rho_{21} = 2\,336\,297\,244\,025\,746$
	$c_{22} = 138\,825\,972\,053\,046$	$c_{22}\rho_{22} = 10\,544\,298\,270\,404\,040.$

Here $c_n\rho_n$ is the sum of the square end-to-end distance of all c_n n-step walks. The mean square end-to-end distance $\langle R_n^2 \rangle = c_n\rho_n/c_n$. For the square lattice, the series for the mean resistance of SAWs with bridges, as described in Manna *et al* (1989), is given by the coefficients: 4.0, 24.0, 90.0, 328.0, 1090.8, 3575.467, 11 156.904, 34 674.676, 104 463.883, 314 446.844, 926 065.313, 2727 633.75, 7907 241.5, 22 961 948.0, 65 989 872.0, 187 568 736.0, 542 658 624.0, 1188 122 112.0 for the coefficients of order 1–17 in the generating function.

2. Analysis of series

The method of analysis used is fully described by Guttmann (1987, 1989a). The data for the triangular lattice permit an extension of the analysis given in Guttmann (1989b). We have constructed both unbiased and biased differential approximants, where the biasing involves assuming that the exponent is $\frac{43}{32}$ exactly for the SAW generating function. Both first and second order differential approximants were used, the results being of comparable quality. Unbiased estimates gave

$x_c = 0.240\,916 \pm 0.000\,0024 \qquad \gamma = 1.3431 \pm 0.0008 \qquad$ (first order)

$x_c = 0.240\,916 \pm 0.000\,0033 \qquad \gamma = 1.3431 \pm 0.0008 \qquad$ (second order).

These results lend strong support to the generally accepted value (Neinhuis 1982) $\gamma = \frac{43}{32} = 1.343\,75$. Accepting this value, linear regression on the approximants gives $x_c = 0.240\,9185 \pm 0.000\,0010$. This may be compared with an earlier estimate (Guttmann 1989b) based on the 20-term series, of $x_c = 0.240\,919$.

The square lattice data was analysed similarly. The unbiased approximants gave the following estimates:

$x_c = 0.379\,0518 \pm 0.000\,0030 \qquad \gamma = 1.343\,55 \pm 0.000\,69 \qquad$ (first order)

$x_c = 0.379\,0519 \pm 0.000\,0012 \qquad \gamma = 1.343\,57 \pm 0.000\,28 \qquad$ (second order).

Biasing the approximants at $\gamma = \frac{43}{32}$ gave $x_c = 0.379\,0526 \pm 0.000\,0005$, which is in good agreement with the estimate based on 56-step polygons (Guttmann and Enting 1988), $x_c = 0.379\,0523 \pm 0.000\,0002$.

As observed in previous studies, the generating function of the mean square end-to-end distance is not as well behaved as the SAW generating function. This remark applies both to the generating function whose coefficients are $c_n\rho_n$ as well as the generating function with coefficients $\langle R_n^2 \rangle$. In both cases a large percentage of defective approximants arise. The advantage of analysing the series for $\langle R_n^2 \rangle$ is that the critical point is of course precisely at 1.0. If we do this, the estimates of the exponent $1+2\nu$ is 2.497 and increasing (first order) and 2.499 and increasing (second order). Alternatively, biasing the generating function for $c_n\rho_n$ at the value of x_c found above gave $\gamma + 2\nu = 2.842 \pm 0.004$. All these results are consistent with the presumably exact value $\nu = \frac{3}{4}$.

The square lattice data gave rise to similar results for $\langle R_n^2 \rangle$, while biased analysis of the generating function for $c_n\rho_n$ at the value of x_c found above gave $\gamma + 2\nu = 2.843 \pm 0.004$. These results are again consistent with the presumably exact value $\nu = \frac{3}{4}$.

Turning now to the resistance data, an unbiased analysis strongly supported the expected belief that the exponent was the same as that of the SAW generating function, plus one. In fact we found $x_c = 0.379\,03 \pm 0.003\,57$ with exponent 2.33 ± 0.33. Subsequent biased analysis, fixing the critical point at the value quoted above gave for the critical exponent 2.340 ± 0.035. From the results of Manna *et al* we would expect this exponent to be $(1+\gamma) = \frac{75}{32} = 2.343\,75$, which is close to our central estimate and well within error bars.

3. Conclusion

We have refined our previous estimates of the critical points and critical exponents of the two-dimensional SAW problem on the square and triangular lattice, on the basis of a significant extension of existing series. We have also given a new series for the resistance of SAWs with bridges, and presented numerical evidence supporting previous conjectures as to the exact exponent values.

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References

Guttmann A J 1987 *J. Phys. A: Math. Gen.* **21** 1839
—— 1989a *Phase Transitions and Critical Phenomena* vol 13, ed C Domb and J Lebowitz (New York: Academic)
—— 1989b *J. Phys. A: Math. Gen.* **22** 2807
Gutmann A J and Enting I G 1988 *J. Phys. A: Math. Gen.* **21** 1165
Manna S S, Guttmann A J and Roy A K 1989 *J. Phys. A: Math. Gen.* **22** 3621
Nienhuis B 1982 *Phys. Rev. Lett.* **49** 1062
Torrie G and Whittington S G 1975 *J. Phys. A: Math. Gen.* **8** 1178
Wang J 1989 *J. Phys. A: Math. Gen.* **22** L969